5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

27-31 October 2019

First Name:

Last Name:

1 Belief functions and uncertainty theories [Didier Dubois]

Please answer the following questions by selecting the appropriate answer, or providing detailed answer as requested.

1.1 How to represent ignorance about a variable $x \in S$?

- ☐ a uniform probability distribution?
- ☑ a uniform possibility distribution?
- ☑ a mass function assigning weight 1 to S?
- ☑ a lower bound 0 on all events $A \subset S$?

1.2 Is it possible to have for a belief function Bel for all $A, B \subseteq S$:

- ☐ $Bel(A \cup B) < \max(Bel(A), Bel(B))$?
  (Violation of monotonicity)
- ☐ $Bel(A \cup B) = \max(Bel(A), Bel(B))$?
  (except with a Dirac (m(s)=1)
- ☐ $Bel(A) + Bel(notA) > 1$?
  (incoherence)
- ☑ $Bel(A \cap B) = \min(Bel(A), Bel(B))$?
  (consonant case)

1.3 Consider a soccer game Firenze vs. Sienna. There are three possible outcomes:

1: F wins, 2: S wins, 3: a draw.

One assigns imprecise probabilities to each outcome

$$p(1) \in [0.1, 0.3]; \quad p(2) \in [0.2, 0.4]; \quad p(3) \in [0.3, 0.5]$$

- Show all bounds are attained by a probability measure in the credal set defined by these probability intervals.

0.1 + 0.2 + 0.5 <= 1 <= 0.3 + 0.4 + 0.3
0.1 + 0.4 + 0.3 <= 1 <= 0.3 + 0.2 + 0.5
0.3 + 0.2 + 0.3 <= 1 <= 0.1 + 0.4 + 0.5

- Compute lower probabilities $P_*(A)$ of all events $\{i\}, \{i,j\}, i \neq j = 1,2,3, \{1,2,3\}$

P*({1, 2}) = max(0.3, 1- 0.5) = 0.5
P*({2, 3}) = max(0.5, 1- 0.3) = 0.7
P*({1, 3}) = max(0.4, 1- 0.4) = 0.6

- Compute the Moebius transform m of P_* (note that $m(\{i\}) = P_*(\{i\})$; then compute $m(\{i,j\})$), then $m(\{1,2,3\})$.

m(1) = 0.1
m(2) = 0.2
m(3) = 0.3
m({1,2}) = 0.5 - 0.1 - 0.2 = 0.2
m({2, 3}) = 0.7 - 0.2 - 0.3 = 0.2
m({1, 3}) = 0.6 - 0.1 - 0.3 = 0.2
m({1, 2, 3}) = 1

Other masses = -0.2

- Is P_* a belief function and why or why not ?

Then no.

1.4 Consider a triangular fuzzy interval (consonant belief function) on the real line with support $[1, 3]$ and core 2. Let

$$\mathcal{P}(\pi) = \{P : P(A) \leq \Pi(A), \forall A\}.$$

Let the p-box limited by the upper and lower probabilities (i.e. possibility and necessity) of $(-\infty, x]$ be denoted by F^* and F_*. Let $\mathcal{P} = \{P : F^* \geq F \geq F_*\}$, where F is the cumulative function for P, be the credal set of the p-box.

Consider the discrete probability measure P such that $P(\{1.5\}) = P(\{2.5\}) = 0.5$.

- Is $P \in \mathcal{P}$?

- What is $P(\{1.5, 2.5\})$? $\Pi(\{1.5, 2.5\})$?

- Is $\mathcal{P}(\pi) = \mathcal{P}$?

A triangular fuzzy number with support [1, 3] and mode 2.
Let P be defined by P({1.5})=P({2.5})=0.5.
Then $F_* < F < F$ but $P \notin P(\Pi)$ since
$P(\{1.5, 2.5\}) = 1 > \Pi(\{1.5, 2.5\}) = 0.5$

