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CENTRE DE MOBILITÉ INTERNATIONALE DE RENNES

Conflict modeling and managment

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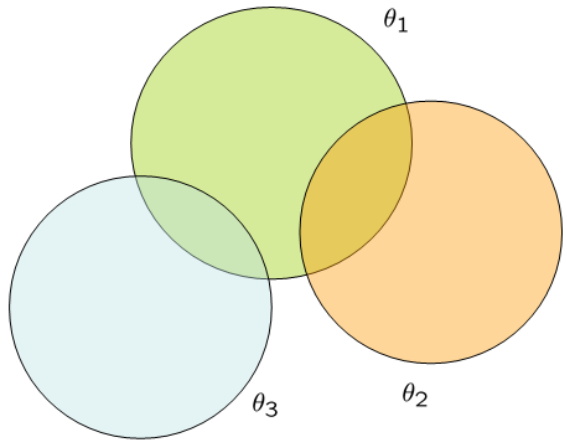
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Notations $^{(1/6)}$

- Discernment frame: $\Theta = \{\theta_1, \ldots, \theta_n\}$, whith θ_i are exclusive and exhaustive classes
- Power set: $2^\Theta = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \{\theta_1 \cup \theta_2\}, \ldots, \Theta\}$.
- Extension of 2^Θ to D^Θ, hyper power set in order to model the conflicts
  - D^Θ closed set by union and intersection operators
  - D_r^Θ: reduced set with constraints $(\theta_2 \cap \theta_3 \equiv \emptyset)$



Conjunctive rules (2/6)

- Assume: two cognitively independent and reliable sources S_1 and S_2.
- The conjunctive rule is given for m_1 and m_2 bbas of S_1 and S_2, for all $X \in 2^\Theta$, with $X \neq \emptyset$ by:

$$m_{\text{Conj}}(X) = \sum_{Y_1 \cap Y_2 = X} m_1(Y_1) m_2(Y_2) \qquad (1)$$

	$\emptyset$	θ_1	θ_2	θ_3	Θ
m_1	0	0.5	0.1	0	0.4
m_2	0	0.2	0	0.5	0.3
m	0.32	0.33	0.03	0.2	0.12

Dempster's rule (3/6)

- Dempster's rule:

$$m_{\mathrm{D}}(X) = \frac{1}{1-k} m_{\mathrm{Conj}}(X) \tag{2}$$

where $k = \sum_{A \cap B = \emptyset} m_1(A) m_2(B)$ is generally called conflict or *global conflict*. That is the sum of the *partial conflicts*.

- That is not a conflict measure (Liu, 2006).
- Conjunctive rules are not idempotent

Idempotence (4/6)

	$\emptyset$	θ_1	θ_2	θ_3	Θ
m_1	0	0.7	0.2	0.1	0
m_2	0	0.7	0.2	0.1	0
m_{D}					
m_{Conj}					

Idempotence (4/6)

	$\emptyset$	θ_1	θ_2	θ_3	Θ
m_1	0	0.7	0.2	0.1	0
m_2	0	0.7	0.2	0.1	0
m_{D}	0	0.91	0.07	0.02	0
m_{Conj}	0.46	0.49	0.04	0.01	0

The mass on the empty set obtained by the combination of two identical bbas is not null.

Discounting (5/6)

From (Shafer, 1976):

$$m_j^\alpha(X) = \alpha_j m_j(X), \forall X \in 2^\Theta$$
$$m_j^\alpha(\Theta) = 1 - \alpha_j(1 - m_j(\Theta))$$

$\alpha_j \in [0, 1]$ discounting coefficient can be seen as the reliability of the source S_j

If $\alpha_j = 0$ the source are completely unreliable, all the mass is transfered on Θ, the total ignorance

The discounting process increases the intervals $[\mathrm{bel}_j, \mathrm{pl}_j]$, and so reduces the global conflict in the conjunctive combination rule.

Confusion in conflict (6/6)

- Many measures are called conflict in the theory of belief functions but are not conflict
- The global conflict contains an indication of the conflict between bbas, but not only
- Some terms such as internal conflict, discord, contradiction are not conflict
- Some uncertainty measures such as entropic measures can be sometimes called conflict, but are not conflict

Outline

1. Uncertainty measures
2. Modeling conflict
3. Managing conflict
4. Decisions with conflicting bbas

Uncertainty measures

- From belief functions (belief, plausibility, etc.), one can define many uncertainty measures
- Klir (1994) makes out
  - the non-specificity measures: quantify the imprecision of a mass function
  - the discord measures: quantify how a mass function is not agree with its self

Non-specificity measures

The non-specificity of a subset X: $\log_2(|X|)$ (Hartley (1928))
This was generalized by Dubois and Prade (1985):

$$\mathrm{NS}(m) = \sum_{X \in 2^\Theta,\ X \neq \emptyset} m(X) \log_2(|X|). \tag{3}$$

If the measure of non-specificity is weak, one can consider the mass function as specific.
Yager (1983) defines a specificity measure by:

$$S(m) = \sum_{X \in 2^\Theta,\ X \neq \emptyset} \frac{m(X)}{|X|}. \tag{4}$$

Uncertainty degree

Problem: the non-specificity measures are define in $[0, \log_2(|\Theta|)]$
Smarandache, Martin and Osswald (2011): non-specificity degree

$$\delta_{\mathrm{NS}}(m) \quad = \quad \sum_{X \in 2^\Theta,\ X \neq \emptyset} m(X) \frac{\log_2(|X|)}{\log_2(|\Theta|)}. \tag{5}$$

We have $\delta_{\mathrm{NS}}(m_\Theta) = 1$, where m_Θ is the categoric mass of total ignorance and $\delta_{\mathrm{NS}}(m) = 0$ for Bayesian masses.
Specificity or Bayesianity degree (*i.e.* the proximity with a probability):

$$\delta_B(m) \quad = \quad 1 - \sum_{X \in 2^\Theta,\ X \neq \emptyset} m(X) \frac{\log_2(|X|)}{\log_2(|\Theta|)}. \tag{6}$$

$\delta_B(m) = 1$ for each Bayesian mass.

Uncertainty degree: illustration

Table: $\Theta = \{\theta_1, \theta_2, \theta_3\}$

	m_1	m_2	m_3	m_4	m_5	m_6	m_Θ
θ_1	0.4	0.3	0.1	0.3	0	0	0
θ_2	0.1	0.1	0.3	0.1	0	0	0
θ_3	0.1	0.1	0.1	0.1	0	0	0
$\theta_1 \cup \theta_2$	0.3	0.3	0.5	0	0.6	0.6	0
$\theta_1 \cup \theta_3$	0.1	0.2	0	0	0.4	0	0
$\theta_2 \cup \theta_3$	0	0	0	0	0	0	0
Θ	0	0	0	0.5	0	0.4	1
δ_B	0.75	0.68	0.68	0.5	0.37	0.23	0
δ_{NS}	0.25	0.32	0.32	0.5	0.63	0.77	1

Discord measures

Some discord measures based on the Shanon (1928) entropy:

$$E(m) = -\sum_{X \in 2^\Theta} m(X) \log_2(\mathrm{pl}(X)), \tag{7}$$

$$C(m) = -\sum_{X \in 2^\Theta} m(X) \log_2(\mathrm{bel}(X)), \tag{8}$$

$$D(m) = -\sum_{X \in 2^\Theta} m(X) \log_2(\mathrm{betP}(X)), \tag{9}$$

with $E(m) \leq D(m) \leq C(m)$.

Discord measures

Shannon entropy on the pignistic probability:

$$-\sum_{X\in 2^\Theta} \mathrm{betP}(X)\log_2(\mathrm{betP}(X)). \tag{10}$$

One can summarize them by:

$$-\sum_{X\in 2^\Theta} m(X)\log_2(1-\mathrm{Con}_m(X)), \tag{11}$$

where Con is called conflict measure of the mass function m on X. That is not a conflict measure but a discord measure

Discord/conflict

Definition *The* **discord** *in the theory of belief functions quantifies how a mass function contradicts itself.*

Definition *The* **conflict** *in the theory of belief functions can be defined by the contradiction between two or more mass functions.*

Auto-conflict: a discord measure

Introduced by Osswald and Martin, 2006, the auto-conflict of order s for one expert is given by:

$$a_s = \left(\textcircled{\cap}_{i=1}^{s} m\right)(\emptyset). \tag{12}$$

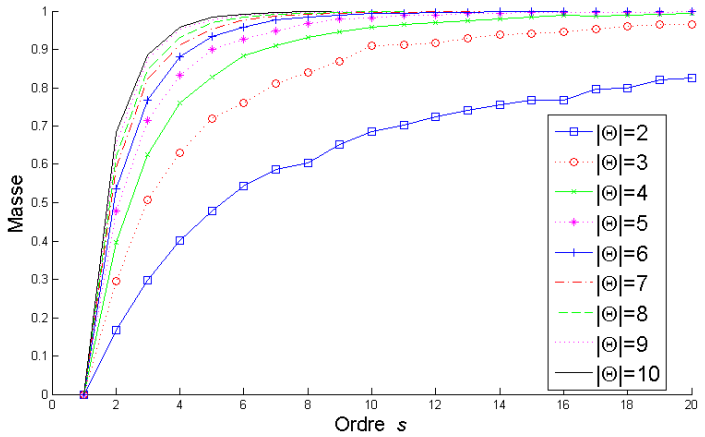
where $\textcircled{\cap}$ is the conjunctive operator.

The following property holds:

$$a_s \leq a_{s+1}$$

$$\lim_{s \to +\infty} a_s = 1 \text{ if } m \text{ is not consonant}$$

The global conflict must be weighted with the auto-conflict.



Mean of auto-conflict for generated bbas

Another discord measure is given by the internal conflict of Schubert (2012).

Outline

1. Uncertainty measures
2. Modeling conflict
3. Managing conflict
4. Decisions with conflicting bbas

Conflict origins

- The sources are not reliable:
  reliability and conflict are very linked in this case
- The discernment frame is not exhaustive:
  the closed world assumption must be avoid
- The sources do not express on the same phenomena:
  bbas must not be combined

Let note $\mathrm{Conf}(m_1, m_2)$ a conflict measure between the mass functions m_1 and m_2.

Conflict measures

- Based on the global conflict coming from the conjunctive rule:

$\mathrm{Conf}(m_1, m_2) = m_{\mathrm{Conj}}(\emptyset)$ (point of view of Smets (2007))
$\mathrm{Conf}(m_1, m_2) = -\ln(1 - m_{\mathrm{Conj}}(\emptyset))$ Yager (1983)

  - pros: $\mathrm{Conf}(m_1, m_\Theta) = 0$
    where $m_\Theta(\Theta) = 1$ is the total ignorance
  - cons:
    1. $\mathrm{Conf}(m_1, m_1) \neq 0$ because of the non-idempotence
    2. $m_{\mathrm{Conj}}(\emptyset)$ contains a part of the auto-conflict of m_1 and m_2
    3. The conjunctive rule needs the independence assumption between the sources: There is no reason to get a link between independence and conflict

Conflict measures

- Based on distance:
  $\mathrm{Conf}(m_1, m_2) = d(m_1, m_2)$ (Martin *et al.*, 2008)
  Conflict between an expert S_j and the $s-1$ other experts is given by the the mean of conflicts two by two:

$$\mathrm{Conf}(m_j, m_{\mathcal{E}}) = \frac{1}{s-1} \sum_{e=1, e\neq j}^{s} \mathrm{Conf}(j, e) \qquad (13)$$

  Another definition is given by: $\mathrm{Conf}(m_j, m_s) = d(m_j, m_{\mathcal{E}})$ where $m_{\mathcal{E}}$ is the bba of the artificial expert build by the combined bbas of the $s-1$ other experts of $\mathcal{E}$ without the expert S_j.
  - pros: $\mathrm{Conf}(m_1, m_1) = 0$
  - cons: $\mathrm{Conf}(m_1, m_\Theta) \neq 0$

Digression on distances

The Jousselme *et al.* (2001) distance can be done by:

$$d(m_1, m_2) = \sqrt{\frac{1}{2}(\mathbf{m}_1 - \mathbf{m}_2)^T \underline{\underline{D}}(\mathbf{m}_1 - \mathbf{m}_2)},$$

where $\underline{\underline{D}}$ is an $2^{|\Theta|} \times 2^{|\Theta|}$ matrix based on Jaccard dissimilarity

$$D(A, B) = \begin{cases} 1, \text{ if } A = B = \emptyset, \\ \\ \dfrac{|A \cap B|}{|A \cup B|}, \forall A, B \in 2^{\Theta}. \end{cases}$$

Distance based on plausibility (Jousselme and Maupin (2011)): $d(m_1, m_2) = 1 - \frac{\mathbf{pl}_1^T . \mathbf{pl}_2}{\|\mathbf{pl}_1\| \|\mathbf{pl}_2\|}$, where **pl** is the plausibility function and $\mathbf{pl}_1^T . \mathbf{pl}_2$ the vector product in 2^n space of both plausibility functions.

Towards efficient conflict measures

Inclusion (Martin (2012))

Definition 1: strict *We say that the mass function* m_1 *is* included *in* m_2 *if all the focal elements of* m_1 *are included in* **all** *focal elements of* m_2*.*
Definition 2: light *We say that the mass function* m_1 *is* included *in* m_2 *if all the focal elements of* m_1 *are included in* **at least one** *focal elements of* m_2*.*
Definition *We note this inclusion by* $m_1 \subseteq m_2$*. The mass functions are* included *if* m_1 *is included in* m_2 *or* m_2 *is included in* m_1*.*

We will define a degree of inclusion.

Axioms

1. **Non-negativity:** $\mathrm{Conf}(m_1, m_2) \geq 0$ A negative conflict does not make sens.
2. **Identity:** $\mathrm{Conf}(m_1, m_1) = 0$
3. **Symmetry:** $\mathrm{Conf}(m_1, m_2) = \mathrm{Conf}(m_2, m_1)$
   The conflict measure must be symmetric.
4. **Normalization:** $0 \leq \mathrm{Conf}(m_1, m_2) \leq 1$
   Not necessary, but the normalization is very useful in many applications of conflict measure.
5. **Inclusion:** $\mathrm{Conf}(m_1, m_2) = 0,$ iif $m_1 \subseteq m_2$ or $m_2 \subseteq m_1$
   This axiom means if the focal elements of two mass functions are not conflicting (the intersection is never empty), the mass functions are not in conflict and the mass functions cannot be in conflict if they are included.

Very similar to the axioms proposed by Destercke and Burger (2012).

Degree of inclusion

- Inclusion index: $Inc(X_1, Y_2) = 1$ if $X_1 \subseteq Y_2$ and 0 otherwise, where X_1 and Y_2 are two focal elements of m_1 and m_2 respectively
- Strict degree of inclusion of m_1 **in** m_2:

$$d_{incS}(m_1, m_2) = \frac{1}{|\mathcal{F}_1||\mathcal{F}_2|} \sum_{X_1 \in \mathcal{F}_1} \sum_{Y_2 \in \mathcal{F}_2} Inc(X_1, Y_2) \qquad (14)$$

  where $\mathcal{F}_1$ and $\mathcal{F}_2$ are the set of focal elements of m_1 and m_2 respectively, and $|\mathcal{F}_1|$, $|\mathcal{F}_2|$ are the number of focal elements of m_1 and m_2.
- Light degree of inclusion of m_1 **in** m_2:

$$d_{incL}(m_1, m_2) = \frac{1}{|\mathcal{F}_1|} \sum_{X_1 \in \mathcal{F}_1} \max_{Y_2 \in \mathcal{F}_2} Inc(X_1, Y_2) \qquad (15)$$

Degree of inclusion

Let $\delta_{inc}(m_1, m_2)$ a **degree of inclusion** of m_1 **and** m_2 define by:

$$\delta_{inc}(m_1, m_2) = \max(d_{inc}(m_1, m_2), d_{inc}(m_2, m_1)) \quad (16)$$

This degree gives the maximum of the proportion of focal elements from one mass function included in another one.

$\delta_{inc}(m_1, m_2) = 1$ if and only if m_1 and m_2 are included, and the axiom of inclusion is reached for $1 - \delta_{inc}(m_1, m_2)$.

Conflict measure

We define a conflict measure between two mass functions m_1 and m_2 by:

$$\text{Conf}(m_1, m_2) = (1 - \delta_{inc}(m_1, m_2))d(m_1, m_2) \tag{17}$$

where d is the distance.
All the previous axioms are reached.
For more than two mass functions, the conflict measure between one expert i and the other $M - 1$ experts can be defined by:

$$\text{Conf}(i, \mathcal{E}) = \frac{1}{M-1} \sum_{j=1, i \neq j}^{M} \text{Conf}(i, j), \tag{18}$$

where $\mathcal{E} = \{1, \ldots, M\}$ is the set of experts in conflict with i.

Illustration

Comportment of the proposed conflict measure

We have: $\mathrm{Conf}(m_1, m_1) = 0$, $\mathrm{Conf}(m_1, m_\Theta) = 0$ and even: if m_1 and m_2 are included then $\mathrm{Conf}(m_1, m_2) = 0$,

Example

$$m_1(\theta_1) = m_1(\theta_2) = m_1(\theta_1 \cup \theta_2 \cup \theta_3) = 1/3,$$

$$m_2(\theta_1 \cup \theta_2) = m_2(\theta_1 \cup \theta_2 \cup \theta_3) = 1/2.$$

On this example, $d(m_1, m_2) = 0.3727$.
$d_{incS}(m_1, m_2) = d_{incL}(m_1, m_2) = 1$ and $d_{incS}(m_2, m_1) = 0.17$, $d_{incL}(m_2, m_1) = 0.5$, therefore $\delta_{inc}(m_1, m_2) = 1$ and $\mathrm{Conf}(m_1, m_2) = 0$

Note:

- $d_{incS}(m_1, m_1) = 0.56$, we only have $d_{incS}(m, m) = 1$ if m is categoric ($m(X) = 1$, $X \in 2^\Theta$).
- $d_{incL}(m_1, m_1) = 1$, $d_{incL}(m, m) = 1 \ \forall m$.

Illustration

- Example:

$$m_3(\theta_3) = m_3(\theta_1 \cup \theta_2 \cup \theta_3) = 0.5.$$

We have $d_{incS}(m_1, m_3) = d_{incS}(m_2, m_3) = 0.5$, but $\mathrm{Conf}(m_1, m_3) = 0.3815$ and $\mathrm{Conf}(m_2, m_3) = 0.3571$. Hence, we obtain:
$\mathrm{Conf}(m_1, m_3) \geq \mathrm{Conf}(m_1, m_2) + \mathrm{Conf}(m_2, m_3)$. m_2 reduce the conflict between m_1 and m_3.
Note: $d_{incL}(m_1, m_3) = d_{incL}(m_2, m_3) = 1$ then $\mathrm{Conf}(m_1, m_3) = \mathrm{Conf}(m_2, m_3) = 0$

- Maximum of conflict: $m_4(\theta_1) = 1$, $m_5(\theta_2) = 1$:
$\mathrm{Conf}(m_4, m_5) = 1$.
That means the most conflicting mass functions are two different categorical mass functions.

Outline

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Reliability estimation

If the conflict comes from unreliable sources: reliability estimation

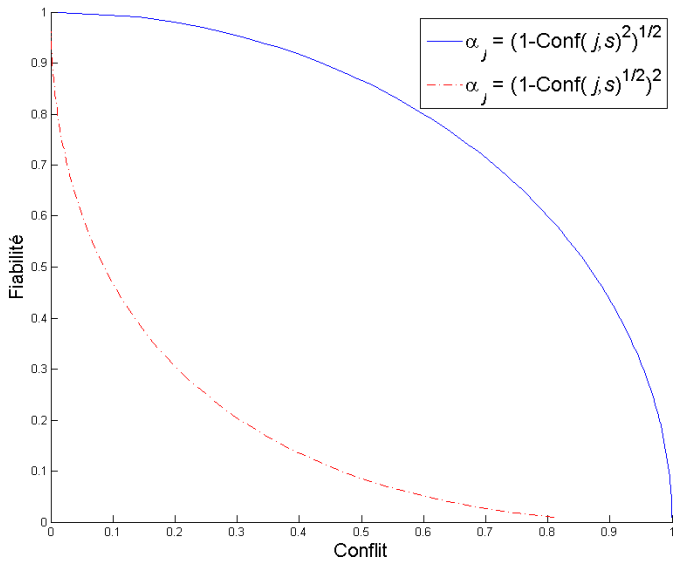
Assumption: a source is unreliable if it is in conflict with the other sources

Reliability measure: such as a decreasing function of the conflict measure

$$\alpha_j = f(\mathrm{Conf}(j,s))$$
$$\alpha_j = \left(1 - \mathrm{Conf}(j,s)^{\lambda}\right)^{1/\lambda}$$

Integration of the reliability measure by discounting

$$m_j^{\alpha}(X) = \alpha_j m_j(X),\ \forall X \in 2^{\Theta}$$
$$m_j^{\alpha}(\Theta) = 1 - \alpha_j(1 - m_j(\Theta))$$



Managing the conflict in the combination

Zadeh example

	$\emptyset$	θ_1	θ_2	θ_3	Θ	Decision
m_1	0	0.9	0	0.1	0	θ_1
m_2	0	0	0.9	0.1	0	θ_2
m_{D}	0	0	0	1	0	θ_3
m_{Conj}	0.99	0	0	0.01	0	θ_3
betP	0	0	0	1	0	θ_3

Solutions

- Suppress the global conflict: disjunctive rule
- Suppress the auto-conflict: idempotent rules
- Assumption of exhaustive discernment frame false: work in open world or add an element
- Transfer the global/partial conflict

Rules transferring the conflict

- Transferring the global conflict
  - Global conflict is coming from the ignorance: Yager's rule
  - Transferring by a weighted function: Inagaki and Lefèvre's rule
  - Change the comportment of the rule according to the global conflict: from the disjunctive to the conjunctive (Florea)

- Transferring the partial conflicts
  - Partial conflict comes from the partial ignorance: Dubois and Prade's rule
  - Proportional redistribution of partial conflict according to the elements that generate it: PCR

Yager (1987) rule

- Assumption: the global conflict comes from the ignorance
- Stay in closed world
- The mass of the empty set is transfered on the global ignorance Θ

$$\begin{aligned} m_{\mathrm{Y}}(X) &= m_{\mathrm{Conj}}(X), \forall X \in 2^{\Theta} \setminus \{\emptyset, \Theta\} \\ m_{\mathrm{Y}}(\Theta) &= m_{\mathrm{Conj}}(\Theta) + m_{\mathrm{Conj}}(\emptyset) \\ m_{\mathrm{Y}}(\emptyset) &= 0. \end{aligned}$$

Inagaki (1991) and Lefèvre (2002) rules

- Stay in closed world
- Propose a global conflict transfer in a general way according to a weighted function.
- The rule is given $\forall X \in 2^\Theta$ by:

$$m_c(X) = m_{\text{Conj}}(X) + w(X) m_{\text{Conj}}(\emptyset),$$

where $\sum_{X \in 2^\Theta} w(X) = 1.$

- The matter is the choose of the thresholds $w(X)$.

Florea (2006) rule

- Stay in closed world
- Propose a global conflict transfer in a such way more the global conflict is high more the rule have a disjunctive comportment.
- The rule is given $\forall X \in 2^\Theta$, $X \neq \emptyset$ by:

$$m_{\text{Flo}}(X) = \beta_1(k) m_{\text{Dis}}(X) + \beta_2(k) m_{\text{Conj}}(X),$$

where β_1 and β_2 have $k = \frac{1}{2}$ like a symmetric weight:

$$\beta_1(k) = \frac{k}{1 - k + k^2},$$
$$\beta_2(k) = \frac{1 - k}{1 - k + k^2}.$$

- We have seen that $1/2$ cannot be the symmetric value of the global conflict
- Other weights are proposed

Dubois and Prade (1988) rule

- Assumption: Partial conflict comes from the partial ignorances
- The partial conflict are transfered on the partial ignorances
- Mixed rule: compromise between conjunctive and disjunctive rules

$$m_{\mathrm{DP}}(X) = \sum_{A \cap B = X} m_1(A) m_2(B) + \sum_{\substack{A \cup B = X \\ A \cap B = \emptyset}} m_1(A) m_2(B).$$

- Precise transfer of the conflict
- Algorithm complexity higher

Mixed rule

- General distribution of partial conflict
- (Martin et Osswald, 2007): **Mixed rule**

$$\begin{aligned} m_{\text{Mix}}(X) &= \sum_{Y_1 \cup Y_2 = X} \delta_1 m_1(Y_1) m_2(Y_2) \\ &+ \sum_{Y_1 \cap Y_2 = X} \delta_2 m_1(Y_1) m_2(Y_2) \end{aligned} \tag{19}$$

- Dubois and Prade's rule
  $\delta_1(Y_1, Y_2) = 1 - \delta_2(Y_1, Y_2) = \mathbb{1}_{Y_1 \cap Y_2 = \emptyset}$
  and Florea's rule can be seen such as a particular case

Mixed rule: taking into account the specificity

The choice of $\delta_1 = 1 - \delta_2$ can be given from a dissimilarity measure such as:

$$\delta_1^1(Y_1, Y_2) = 1 - \frac{|Y_1 \cap Y_2|}{\min(|Y_1|, |Y_2|)} \tag{20}$$

or from the Jaccard's dissimilarity:

$$\delta_1^2(Y_1, Y_2) = 1 - \frac{|Y_1 \cap Y_2|}{|Y_1 \cup Y_2|} \tag{21}$$

- If $Y_1 \cap Y_2 = \emptyset$: partial conflicts can be interpreted such as partial ignorances
- If $Y_1 \cap Y_2 \notin \{Y_1, Y_2, \emptyset\}$: transfer on $Y_1 \cap Y_2$ and $Y_1 \cup Y_2$ according to δ_1^1 and δ_1^2

The PCR6 (Martin et Osswald, 2006, 2007)

This rule transfers the partial conflicts on the elements that generate it, proportionally to their masses.

$$m_{\mathrm{PCR6}}(X) = m_{\mathrm{Conj}}(X) + \sum_{\substack{Y\in 2^{\Theta},\\ X\cap Y=\emptyset}} \left(\frac{m_1(X)^2 m_2(Y)}{m_1(X)+m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X)+m_1(Y)} \right) \quad (22)$$

$$m_{\mathrm{PCR6}}(X) = m_{\mathrm{Conj}}(X) + \sum_{j=1}^{s} m_i(X)^2 \sum_{\substack{\bigcap_{j'=1}^{s-1} Y_{\sigma_j(j')}\cap X=\emptyset \\ (Y_{\sigma_j(1)},\ldots,Y_{\sigma_j(s-1)})\in(2^{\Theta})^{s-1}}} \left(\frac{\prod_{j'=1}^{s-1} m_{\sigma_j(j')}(Y_{\sigma_j(j')})}{m_j(X)+\sum_{j'=1}^{s-1} m_{\sigma_j(j')}(Y_{\sigma_j(j')})} \right)$$

	$\emptyset$	θ_1	θ_2	$\theta_1\cup\theta_2$	θ_3	$\theta_1\cup\theta_3$	$\theta_2\cup\theta_3$	Θ	Decision
m_{D}	0	0	0	0	1	0	0	0	θ_3
m_{Conj}	0.99	0	0	0	0.01	0	0	0	θ_3
m_{DP}	0	0	0	0.81	0.01	0.09	0.09	0	θ_3
m_{PCR6}	0	0.486	0.486	0	0.028	0	0	0	θ_1 ou θ_2

Taking into account a local reliability

(Martin and Osswald, 2007): **discounting PCR, DPCR**
Redistribution of the partial conflict proportionally on elements implied in the partial conflict and on the partial ignorance according to a discounting factor: $\alpha \in [0, 1]$.

$$\begin{aligned} m_{\mathrm{DPCR}}(X) = m_{\mathrm{Conj}}(X) \\ + \sum_{\substack{Y \in 2^\Theta, \\ X \cap Y = \emptyset}} \alpha \left(\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)} \right) \\ + \sum_{\substack{Y_1 \cup Y_2 = X \\ Y_1 \cap Y_2 = \emptyset}} (1 - \alpha) m_1(Y_1) m_2(Y_2) \end{aligned} \tag{23}$$

- If $Y_1 \cap Y_2 \neq \emptyset$: conjunctive comportment
- If $Y_1 \cap Y_2 = \emptyset$: transfer on Y_1, Y_2 and $Y_1 \cup Y_2$

Estimation of the local reliability

Local conflict: defined on $(2^{\Theta})^s$ and takes values in $\left[0, \frac{1}{s}\right]$

$$f_j(Y_1, ..., Y_s) = \frac{\sum_{j'=1}^{s} \mathbb{1}_{\{\emptyset\}}(Y_{j'} \cap Y_j)}{s(s-1)} \quad (24)$$

Local reliability:

$$\alpha(Y_1, ..., Y_s) = 1 - \sum_{j=1}^{s} f_j(Y_1, ..., Y_s) \quad (25)$$

Estimation of the local reliability

Local conflict: defined on $(2^{\Theta})^s$ and takes values in $\left[0, \frac{1}{s}\right]$

$$f_j(Y_1, ..., Y_s) = \frac{\sum_{j'=1}^{s} \mathbb{1}_{\{\emptyset\}}(Y_{j'} \cap Y_j)}{s(s-1)} \quad (24)$$

Local non-conflict: defined on $(2^{\Theta})^s$ and takes values in $\left[0, \frac{1}{s}\right]$

$$\alpha_j(Y_1, ..., Y_s) = \frac{1}{s} - f_j(Y_1, ..., Y_s) = \frac{\sum_{j'=1, j' \neq j}^{s} \mathbb{1}_{\{\emptyset\}^c}(Y_{j'} \cap Y_j)}{s(s-1)} \quad (25)$$

Taking into account reliability and specificity

(Martin et Osswald, 2007): **mixed** and **discounting** PCR, MDPCR

$$
\begin{aligned}
m_{\mathrm{MDPCR}}(X) = & \sum_{\substack{Y_1 \cup Y_2 = X, \\ Y_1 \cap Y_2 \neq \emptyset}} \delta\, m_1(Y_1) m_2(Y_2) \\
& + \sum_{\substack{Y_1 \cap Y_2 = X, \\ Y_1 \cap Y_2 \neq \emptyset}} (1-\delta) m_1(Y_1) m_2(Y_2) \\
& + \sum_{\substack{Y \in 2^\Theta, \\ X \cap Y = \emptyset}} \alpha \left(\frac{m_1(X)^2 m_2(Y)}{m_1(X)+m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X)+m_1(Y)} \right) \\
& + \sum_{\substack{Y_1 \cup Y_2 = X \\ Y_1 \cap Y_2 = \emptyset}} (1-\alpha) m_1(Y_1) m_2(Y_2)
\end{aligned}
\tag{26}
$$

- If $Y_1 \cap Y_2 = \emptyset$: transfer on Y_1, Y_2 et $Y_1 \cup Y_2$
- If $Y_1 \cap Y_2 \notin \{Y_1, Y_2, \emptyset\}$: transfer on $Y_1 \cap Y_2$ and $Y_1 \cup Y_2$

What to do to manage the conflict?

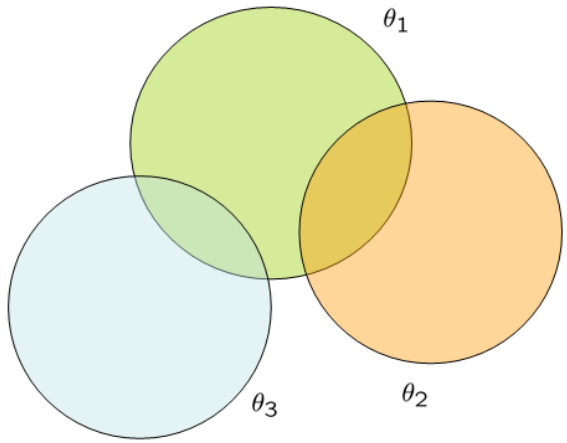
- First, if you have any reliable information on the reliability of the sources: discount the bas.
- If there is still global conflict: choose an appropriate rule to manage this conflict.
- There is no an optimal rule local for any application.
  1. Choose the rule according to the needed properties (reliability, idempotence, independence, complexity, etc.)
  2. Try some rules and choose the best for your application

Outline

1. Uncertainty measures
2. Modeling conflict
3. Managing conflict
4. Decisions with conflicting bbas

Decisions with conflicting bbas

- Another point of view: do not manage the conflict and keep it until the decision
- That is the point of view of Smets with the use of the conjunctive rule and the pignistic probability for the decision.
- Other idea: keep all the partial conflicts during the combination rule:
  - Hyper power set $D^\Theta = \{\theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$, if $\Theta = \{\theta_1, \theta_2\}$
  - D_r^Θ: reduced set with constraints $(\theta_2 \cap \theta_3 \equiv \emptyset)$



Decision

- In general the decision is made on Θ and not on 2^Θ
  - Pessimist: $\max_{\theta \in \Theta} bel(\theta)$
  - Optimist: $\max_{\theta \in \Theta} pl(\theta)$
  - Compromise: $\max_{\theta \in \Theta} betP(\theta)$

Pignistic probability:

$$\mathrm{betP}(X) = \sum_{Y \in 2^\Theta, Y \neq \emptyset} \frac{|X \cap Y|}{|Y|} \frac{m(Y)}{1 - m(\emptyset)} \tag{27}$$

$$\mathrm{GPT}(X) = \sum_{Y \in D_r^\Theta, Y \neq \emptyset} \frac{\mathcal{C}_\mathcal{M}(X \cap Y)}{\mathcal{C}_\mathcal{M}(Y)} m(Y) \tag{28}$$

where $\mathcal{C}_\mathcal{M}(X)$ is the cardinality X of D_r^Θ

Decision on the unions

- Decision on 2^Θ
- The decision functions f_d (belief, plausibility, pignistic probability, etc.) increase by inclusion (Appriou, 2005)

$$A = \underset{X \in 2^\Theta}{\operatorname{argmax}} \left(m_d(X) f_d(X) \right),$$

where

$$m_d(X) = \left(\frac{K_d \lambda_X}{|X|^r} \right)$$

with $r \in [0, 1]$, is a weighted factor of the waited precision of the decision:
$r = 1$: singleton,
$r = 0$: ignorance

Decision process

(Martin et Quidu, 2008) decision process:

1. Reject the elements not in the learned classes

   (Le Hégarat-Mascle *et al.* 1997)

$$\begin{cases} \text{bel}(\theta_d) = \max\limits_{1 \leq i \leq n} \text{bel}(\theta_i) \\ \text{bel}(\theta_d) \geq \text{bel}(\theta_d^c) \end{cases} \tag{29}$$

2. Decision on 2^Θ on the non-rejected elements

   The decision functions f_d (belief, plausibility, pignistic probability, etc.) increase by inclusion (Appriou, 2005)

$$A = \underset{X \in 2^\Theta}{\text{argmax}} \left(m_d(X) f_d(X) \right), \quad m_d(X) = \left(\frac{K_d \lambda_X}{|X|^r} \right) \tag{30}$$

$r \in [0, 1]$, $r = 1$: singleton, $r = 0$: ignorance

Decision in presence of conflict

Example



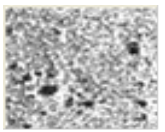
Data GESMA



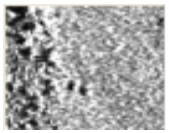
Sand



Rock



Rock OR Sand



Rock AND Sand

Wreck or object

Decision on conflicting bbas

(Martin, 2008): decision on D^Θ

- on D_r^Θ

$$A = \underset{X \in D_r^\Theta}{\operatorname{argmax}} \left(m_d(X) f_d(X) \right)$$

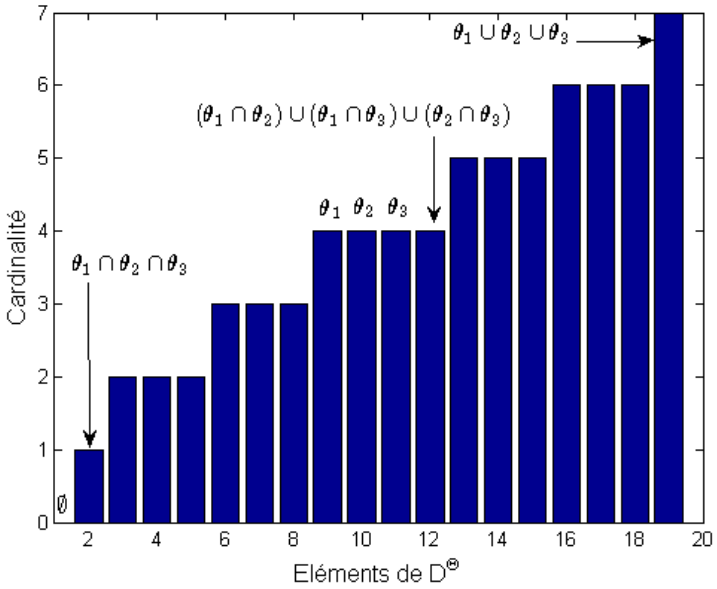
$$m_d(X) = \left(\frac{K_d \lambda_X}{\mathcal{C}_\mathcal{M}(X)^r} \right)$$

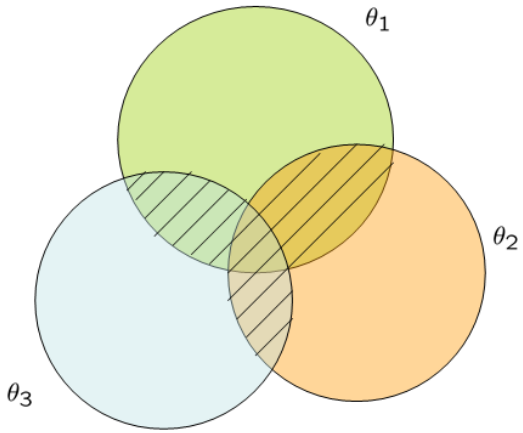
- on $\mathcal{D}$ a subset of D^Θ

$$A = \underset{X \in \mathcal{D}}{\operatorname{argmax}} \left(m_d(X) f_d(X) \right)$$

such as the cardinality

$$\mathcal{D} = \left\{ X \in D_r^\Theta ; \min_C \leq \mathcal{C}_\mathcal{M}(X) \leq \max_C \right\}$$





To end

Take care of conflict

- Model the conflict correctly
- Suppress the conflict
- Manage the conflict in the combination rule
- Keep the conflict until the decision step

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