5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

27-31 October 2019



First Name:
Last Name:

1 Introduction to belief functions [Thierry Denœux]

1.1 Exercise 1

An urn contains 90 balls, of which 30 are white, and 60 are either black or yellow. A ball is going to be drawn from the urn. Represent the uncertainty about the outcome of this experiment using a mass function on a suitable frame. Compute the corresponding belief and plausibility functions.

1.2 Exercise 2

Let $\Omega = \{a, b, c\}$ and f the following function from 2^Ω to $[0, 1]$:

A	$\varnothing$	$\{a\}$	$\{b\}$	$\{a, b\}$	$\{c\}$	$\{a, c\}$	$\{b, c\}$	$\{a, b, c\}$
$f(A)$	0	0.5	0.2	0.8	0	0.5	0.5	1

Is f a belief function?

1.3 Exercise 3

Let $\Omega = \{a, b\}$, and let m and m' be the following mass functions on Ω,

$$m = \{a\}^\alpha \oplus \{b\}^\beta, \quad m' = \{a\}^{\alpha'} \oplus \{b\}^{\beta'},$$

where A^w denotes the simple mass function m with weight of evidence w on A.

1. Compute m and m'.
2. Compute $m \oplus m'$.

5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

27-31 October 2019



First Name:
Last Name:

2 Belief functions and Boolean inference [Sébastien Destercke]

Please answer the following questions by selecting the appropriate answer, or providing detailed answer as requested.

2.1 If I have n propositional variables, my number of possible focal elements (subsets of interpretations) is

☐ n ☐ 22^n ☐ 42

☐ $n!$ ☐ 2^{2^n}

2.2 Which of the following tools/methods is tailored to check whether a knowledge base (set of logical formulas) is consistent?

☐ SAT solvers ☐ Simulated annealing

☐ Navier-Stokes equations ☐ Pizza oven

2.3 The minimum of a multi-linear function whose variables are constrained to belong to an hyper-cube is obtained

☐ on one of the vertex of the hyper-cube ☐ within the hyper-cube

☐ on the lower bounds of each dimension of the hyper-cube ☐ by sheer luck

2.4 A particular field of application where one can encounter monotone Boolean functions is

- ☐ Quantum mechanics
- ☐ Reliability analysis
- ☐ French Literature
- ☐ Non-linear control

5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

27-31 October 2019

First Name:
Last Name:

3 Belief functions and uncertainty theories [Didier Dubois]

Please answer the following questions by selecting the appropriate answer, or providing detailed answer as requested.

3.1 How to represent ignorance about a variable $x \in S$?

☐ a uniform probability distribution? ☐ a uniform possibility distribution?

☐ a mass function assigning weight 1 to S? ☐ a lower bound 0 on all events $A \subset S$?

3.2 Is it possible to have for a belief function Bel for all $A, B \subseteq S$:

☐ $Bel(A \cup B) < \max(Bel(A), Bel(B))$? ☐ $Bel(A \cup B) = \max(Bel(A), Bel(B))$?

☐ $Bel(A) + Bel(notA) > 1$? ☐ $Bel(A \cap B) = \min(Bel(A), Bel(B))$?

3.3 Consider a soccer game Firenze vs. Sienna. There are three possible outcomes:

1: F wins, 2: S wins, 3: a draw.

One assigns imprecise probabilities to each outcome

$$p(1) \in [0.1, 0.3]; \quad p(2) \in [0.2, 0.4]; \quad p(3) \in [0.3, 0.5]$$

- Show all bounds are attained by a probability measure in the credal set defined by these probability intervals.

- Compute lower probabilities $P_*(A)$ of all events $\{i\}, \{i,j\}, i \neq j = 1,2,3, \{1,2,3\}$

- Compute the Moebius transform m of P_* (note that $m(\{i\}) = P_*(\{i\})$; then compute $m(\{i,j\})$), then $m(\{1,2,3\})$.

- Is P_* a belief function and why or why not ?

3.4 Consider a triangular fuzzy interval (consonant belief function) on the real line with support $[1, 3]$ and core 2. Let

$$\mathcal{P}(\pi) = \{P : P(A) \leq \Pi(A), \forall A\}.$$

Let the p-box limited by the upper and lower probabilities (i.e. possibility and necessity) of $(-\infty, x]$ be denoted by F^* and F_*. Let $\mathcal{P} = \{P : F^* \geq F \geq F_*\}$, where F is the cumulative function for P, be the credal set of the p-box.

Consider the discrete probability measure P such that $P(\{1.5\}) = P(\{2.5\}) = 0.5$.

- Is $P \in \mathcal{P}$?

- What is $P(\{1.5, 2.5\})$? $\Pi(\{1.5, 2.5\})$?

- Is $\mathcal{P}(\pi) = \mathcal{P}$?

5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

27-31 October 2019



First Name:
Last Name:

4 Rough sets and belief functions [Davide Ciucci]

Please answer the following questions by selecting the appropriate answer, or providing detailed answer as requested.

4.1 Which of the following one is true?

☐ The upper approximation is always contained in the lower approximation

☐ An approximation space is a not-well known set of objects

☐ The boundary of a set can be empty

☐ A rough set can be defined only by an equivalence relation

4.2 A reduct in rough set theory is used for the following machine learning task:

☐ Clustering

☐ Feature selection

☐ Regression

☐ Anomaly detection

4.3 An uncertain decision table is

☐ A decision table where the decision attribute is not exactly known, and the uncertainty is expressed in the form of a basic belief assignment on the possible values

☐ A decision table with missing values

☐ A decision table where the condition attributes (features) are not exactly known, and the uncertainty is expressed in the form of a basic belief assignment on the possible values

☐ A set-valued decision table

4.4 Which is the relationship between rough clustering and credal clustering?

☐ None

☐ They are equivalent

☐ Rough Clustering is a special case of credal clustering

☐ Credal clustering is a special case of rough clustering

5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

27-31 October 2019



First Name:
Last Name:

5 Methods for building belief functions [Thierry Denœux]

5.1 Exercise 1

Let $D = \{d_1, d_2\}$ be a set of diseases and $S = \{s_1, s_2, s_3\}$ a set of symptoms. We have the following conditional mass functions:

$$m^S(\{s_1\} \mid d_1) = 0.3, \quad m^S(\{s_1, s_2\} \mid d_1) = 0.5, \quad m^S(S \mid d_1) = 0.2$$

$$m^S(\{s_2\} \mid d_2) = 0.4, \quad m^S(\{s_2, s_3\} \mid d_2) = 0.5, \quad m^S(S \mid d_2) = 0.1$$

1. Compute the conditional embeddings of these mass functions in $D \times S$, and their orthogonal sum.

2. What can we say about the disease if we observe symptom s_i, $i = 1, 2, 3$?

3. What can we say about the disease if we observe that the symptom is either s_i or s_j, $i, j = 1, 2, 3$, $i \neq j$?

5.2 Exercise 2

We consider the following mass functions on $\Omega = \{a, b, c\}$: $m(\{a\}) = 0.3$, $m(\{a, b\}) = 0.3$, $m(\{b, c\}) = 0.4$. Compute the contextual discounting of m with $\Theta = \{\{a\}, \{b, c\}\}$, $\alpha_1 = 0.5$, $\alpha_2 = 0.7$.

5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

27-31 October 2019



First Name:
Last Name:

6 Information correction and fusion using belief functions [Frédéric Pichon]

Please answer the following questions by selecting the appropriate answer, or providing detailed answer as requested.

6.1 Let X be a variable with possible values in $\mathcal{X} = \{a, g, h\}$. Let m_1 and m_2 be two simple mass functions on $\mathcal{X}$ such that:

$$m_1(\{a, g\}) = 0.7, m_1(\mathcal{X}) = 0.3$$
$$m_2(\{a, h\}) = 0.6, m_2(\mathcal{X}) = 0.4$$

Let m be the mass function on $\mathcal{X}$ resulting from the conjunctive combination of m_1 and m_2, with dependence structure characterized by parameter σ such that $\sigma = 0.1$. m is defined by

- ☐ $m(\{a\}) = 0.52, m(\{a, g\}) = 0.08, m(\{a, h\}) = 0.18, m(\mathcal{X}) = 0.22$
- ☐ $m(\{a\}) = 0.52, m(\{a, h\}) = 0.08, m(\{a, g\}) = 0.18, m(\mathcal{X}) = 0.22$
- ☐ $m(\{a\}) = 0.42, m(\{a, g\}) = 0.18, m(\{a, h\}) = 0.28, m(\mathcal{X}) = 0.12$

6.2 Let X be a variable with possible values in $\mathcal{X} = \{a, g, h\}$. Let m_S be an uncertain piece of information about the actual value of X provided by a source S, such that:

$$m_S(\{a, g\}) = 0.8, m_S(\{h\}) = 0.2$$

Assuming that S is relevant with probability 0.9 and truthful with probability 0.6, and that its relevance is independent from its truthfulness, the induced knowledge about X is represented by the mass function m such that

- ☐ $m(\{a\}) = 0.252, m(\{g\}) = 0.252, m(\{h\}) = 0.396, m(\mathcal{X}) = 0.1$

- ☐ $m(\{a, g\}) = 0.72, m(\{g\}) = 0.18, m(\mathcal{X}) = 0.1$
- ☐ $m(\{a, g\}) = 0.504, m(\{h\}) = 0.396, m(\mathcal{X}) = 0.1$

6.3 Let $\odot$ denote the combination rule defined for any two mass functions m_1 and m_2 by

$$(m_1 \odot m_2)(A) = \sum_{(B \cap \overline{C}) \cup (\overline{B} \cap C) \cup (\overline{B} \cap \overline{C}) = A} m_1(B) m_2(C), \quad \forall A \subseteq \mathcal{X}.$$

Which set of assumptions on the sources providing m_1 and m_2 induce this rule ?

- ☐ the sources are independent, relevant, and at least one of them is not truthful
- ☐ the sources are independent, truthful, and at least one of them is not relevant

5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

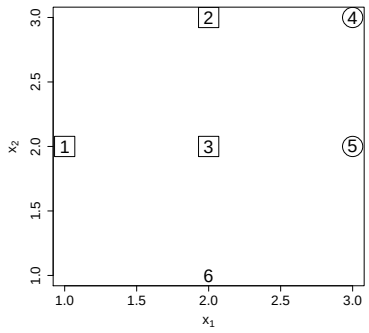
27-31 October 2019

First Name:
Last Name:

7 Classification and clustering using belief functions [Thierry Denœux]

7.1 Exercise 1

We consider a classification problem with two classes $\Omega = \{\omega_1, \omega_2\}$ and two attributes and the following dataset:



In this exercise, we use the evidential K-nearest neighbor rule with $K = 3$ and functions φ_1 and φ_2 taking the following values:

d	0	1	$\sqrt{2}$	$+\infty$
$\varphi_1(d)$	1	0.5	0.2	0
$\varphi_2(d)$	1	0.7	0.5	0

We wish to classify vector 6. Compute the corresponding mass function.

7.2 Exercise 2

Show that, if the K nearest neighbors of a data point x are located at the same distance of x, and if $\varphi_1 = \varphi_2 = \ldots = \varphi_c = \varphi$, then the evidential K-NN rule yields the same decision as the voting K-NN rule.

5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

27-31 October 2019



First Name:
Last Name:

8 Graphical models for belief functions [Prakash Shenoy]

Please answer the following questions by selecting the appropriate answer, or providing detailed answer as requested.

8.1 There are several equivalent mathematical representations of belief functions. Which one of the below is semantically inappropriate for the Dempster-Shafer theory of belief functions:

☐ basic probability assignment (aka mass functions)

☐ plausibility functions

☐ commonality functions

☐ credal sets of probability mass functions

8.2 Suppose m_1 is a mass function for A, and m_2 is a mass function for $\{A, B\}$, and m_3 is a mass function for $\{A, C\}$. Assume these are all distinct and we wish to find the marginal of the joint for C. Which of the following is the most efficient and correct method for finding such a marginal:

☐ $(m_1 \oplus m_2 \oplus m_3)^{\downarrow C}$

☐ $m_3^{\downarrow C}$

☐ $(m_1 \oplus m_2^{\downarrow A} \oplus m_2)^{\downarrow C}$

☐ $((m_1 \oplus m_2)^{\downarrow A} \oplus m_3)^{\downarrow C}$

8.3 There are many combination rules for combining belief functions. Which of the following are central to the Dempster-Shafer theory of belief functions:

☐ Shafer's rule for discounting belief functions

☐ Dempster's rule for combining belief functions

☐ Disjunctive rule for combining belief functions

☐ Fagin-Halpern rule for updating belief functions

8.4 The D-S theory of belief functions is an attractive uncertainty calculus for reasoning under uncertainty because:

☐ It is more efficient than using probability theory

☐ It is more flexible than using probability theory when we have missing information about priors and/or conditionals

☐ It is a newer theory that is more modern than probability theory

☐ It is more efficient than using possibility theory

8.5 Graphical methods have become popular in practice because

☐ They allow us to represent knowledge in small chunks and reason from them

☐ They allow us to build very large models

☐ We can compute marginals of the joint very efficiently without explicitly computing the joint

☐ All of the above

5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

27-31 October 2019



First Name:
Last Name:

9 Distances and conflict between belief functions [Anne-Laure Jousselme]

Please answer the following questions by selecting the appropriate answer, or providing detailed answer as requested.

9.1 Which property is not satisfied by Dempster's conflict, $\kappa(m_1, m_2) = (m_1 \textcircled{\cap} m_2)(\varnothing)$?

☐ Symmetry ☐ Positivity

☐ Reflexivity ☐ Imprecision monotonicity

9.2 Let $X = \{x_1, x_2, x_3\}$ be a frame of discernment and let us consider the two following dichotomous belief functions defined over X:

$m_1(\{x_2\}) = 0.32$	$m_2(\{x_2\}) = 0.49$
$m_1(\{x_1, x_3\}) = 0.15$	$m_2(\{x_1, x_3\}) = 0.49$
$m_1(X) = 0.53$	$m_2(X) = 0.02$

We have that: $d_J^{(2)}(m_1, m_2) = 0.34$ and $d_{Bet}^{(2)}(m_1, m_2) = 0$. Which metric property is *obviously* not satisfied by $d_{Bet}^{(2)}$?

☐ Reflexivity ☐ Triangle inequality

☐ Separability ☐ Symmetry

9.3 Let $X = \{x_1, x_2\}$ be a frame of discernment and let us consider the two following BPAs defined over X:

$$m_1 = \begin{pmatrix} 0.1 \\ 0 \\ 0.9 \end{pmatrix} \quad m_2 = \begin{pmatrix} 0.2 \\ 0 \\ 0.8 \end{pmatrix}$$

What are the values of $d_I^{(2)}(m_1, m_2)$ and $d_J^{(2)}(m_1, m_2)$?

☐ $\begin{cases} d_I^{(2)}(m_1, m_2) = 0.2 \\ d_J^{(2)}(m_1, m_2) = 0.2 \end{cases}$

☐ $\begin{cases} d_I^{(2)}(m_1, m_2) = 0 \\ d_J^{(2)}(m_1, m_2) = 0.1 \end{cases}$

☐ $\begin{cases} d_I^{(2)}(m_1, m_2) = 0.14 \\ d_J^{(2)}(m_1, m_2) = 0.10 \end{cases}$

☐ $\begin{cases} d_I^{(2)}(m_1, m_2) = 0.51 \\ d_J^{(2)}(m_1, m_2) = 0.65 \end{cases}$

9.4 Explain the difference of the results obtained.

5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

27-31 October 2019



First Name:
Last Name:

10 Belief functions in decision theory [Barbara Vantaggi]

Please answer the following questions by selecting the appropriate answer, or providing detailed answer as requested.

10.1 Expected value models satisfy the independence axiom

☐ On every set of acts

☐ Only in the sets of comonotonic acts

☐ For all sets of acts except the comonotonic ones

☐ Never

10.2 Choquet expected value models with respect to belief functions satisfy the independence axiom

☐ On every set of acts

☐ On sets of comonotonic acts

☐ For all sets of acts except the comonotonic ones

☐ Never

10.3 Given

$$\begin{pmatrix} \{0\} & \{100\} & \{0,100\} \\ \frac{1}{4} & \frac{1}{4} & 1 \end{pmatrix}$$

Establish whether it is a generalized lottery (that is the measure over the prize set is a belief function) and determine its Choquet expected value.

☐ The uncertainty measure is not a belief function

☐ The uncertainty measure is a capacity and the Choquet expected value is 20

☐ The uncertainty measure is a belief function and the Choquet expected value is 25

☐ The uncertainty measure is a belief function and the Choquet expected value is 20

10.4 Consider the following generalized lotteries over the prize set, $X = \{10, 100\}$

$$A = \begin{pmatrix} \{10\} & \{100\} & \{10,100\} \\ \frac{1}{4} & \frac{3}{4} & 1 \end{pmatrix} \qquad B = \begin{pmatrix} \{10\} & \{100\} & \{10,100\} \\ \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} \{10\} & \{100\} & \{10,100\} \\ 0 & \frac{2}{3} & 1 \end{pmatrix} \qquad D = \begin{pmatrix} \{10\} & \{100\} & \{10,100\} \\ 0 & \frac{3}{4} & 1 \end{pmatrix}$$

Establish which relation is rational (with respect to a Choquet expected utility)

☐ It is never rational

☐ $B \prec A$ and $D \prec C$

☐ $B \prec A$ and $C \prec D$

☐ $B \preceq A$ and $D \prec C$

5th school on Belief Functions and Their Applications

La Certosa di Pontignano, Siena, Italy

27-31 October 2019

First Name:
Last Name:

11 Multi-criteria decision-making support with belief functions [Jean Dezert]

Please answer the following questions by selecting the appropriate answer, or providing detailed answer as requested.

11.1 Which of the following probabilistic transformations of a belief mass m can be out of the belief interval bounds $[Bel, Pl]$?

- ☐ P_{Pl} transformation
- ☐ $BetP$ transformation
- ☐ P_m transformation
- ☐ $DSmP$ transformation

11.2 Consider a frame of discernment $\Theta = \{\theta_1, \ldots, \theta_n\}$ and suppose that there exists a $\theta_i \in \Theta$ such that $Bel(\theta_i) > 0$.

Can the probabilistic transformation $P_{Bel}(\theta_i) = \frac{Bel(\theta_i)}{\sum_{\theta_i \in \Theta} Bel(\theta_i)}$ be out of the belief interval bounds $[Bel(\theta_i), Pl(\theta_i)]$?

- ☐ Yes
- ☐ No

Hint: $\Theta = \{A, B, C\}$ and $m(A) = 0.8$ and $m(B \cup C) = 0.2$

11.3 What is the weighting vector of OWA approach corresponding to pessimistic decisional attitude?

- ☐ $\mathbf{w} = [1\ 0 \ldots 0\ 0]$
- ☐ $\mathbf{w} = [0\ 0 \ldots 0\ 1]$
- ☐ $\mathbf{w} = [0.5\ 0 \ldots 0\ 0.5]$

11.4 Is there an MCDM method developed so far robust to rank reversal?

☐ Yes ☐ No

11.5 What is the total mass of disagreement between two criteria C_j and $C_{j'}$ relatively to a given hypothesis A_i?

☐ $m_{ij}(A_i)m_{ij'}(A_i) + m_{ij}(\bar{A}_i)m_{ij'}(\bar{A}_i)$ ☐ $m_{ij}(A_i)m_{ij'}(\bar{A}_i) + m_{ij}(\bar{A}_i)m_{ij'}(A_i)$

☐ $m_{ij}(\bar{A}_i)m_{ij'}(A_i)$ ☐ $m_{ij}(A_i)m_{ij'}(\bar{A}_i)$

11.6 Can we always simplify an MCDM problem by the Inter-Criteria analysis?

☐ Yes ☐ No